

Rare radiative B decay to the orbitally excited $K_2^*(1430)$ mesonD. Ebert¹, R. N. Faustov², V. O. Galkin² and H. Toki³¹*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567, Japan
and Institut für Physik, Humboldt-Universität zu Berlin, 10115 Berlin, Germany*²*Russian Academy of Sciences, Scientific Council for Cybernetics, Moscow 117333, Russia*³*Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567, Japan*

Abstract

The exclusive rare radiative B meson decay to the orbitally excited tensor $K_2^*(1430)$ meson is investigated in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. The calculated branching ratio $BR(B \rightarrow K_2^*(1430)\gamma) = (1.7 \pm 0.6) \times 10^{-5}$ as well as the ratio $BR(B \rightarrow K_2^*(1430)\gamma)/BR(B \rightarrow K^*(892)\gamma) = 0.38 \pm 0.08$ is found in a good agreement with recent experimental data from CLEO.

Rare radiative decays of B mesons represent an important test of the standard model of electroweak interactions. These transitions are induced by flavour changing neutral currents and thus they are sensitive probes of new physics beyond the standard model. Such decays are governed by one-loop (penguin) diagrams with the main contribution from a virtual top quark and a W boson. The statistics of rare radiative B decays considerably increased since the first observation of the $B \rightarrow K^*\gamma$ decay in 1993 by CLEO [1]. This allowed a significantly more precise determination of exclusive and inclusive branching ratios [2]. Recently the first observation of rare B decay to orbitally excited tensor strange meson $B \rightarrow K_2^*(1430)\gamma$ has been reported by CLEO [2] with a branching ratio

$$BR(B \rightarrow K_2^*(1430)\gamma) = (1.66_{-0.53}^{+0.59} \pm 0.13) \times 10^{-5}, \quad (1)$$

as well as the ratio of exclusive branching ratios

$$r \equiv \frac{BR(B \rightarrow K_2^*(1430)\gamma)}{BR(B \rightarrow K^*(892)\gamma)} = 0.39_{-0.13}^{+0.15}. \quad (2)$$

These new experimental data provide a challenge to the theory. Many theoretical approaches have been employed to predict the exclusive $B \rightarrow K^*(892)\gamma$ decay rate (for a review see [3] and references therein). Less attention has been paid to rare radiative B decays to excited strange mesons [4–6]. Most of these theoretical approaches [5,6] rely on the heavy quark limit both for the initial b and final s quark and the nonrelativistic quark model. However, the two predictions [5,6] for the ratio r in Eq. (2) differ by an order of magnitude, due to a different treatment of the long distance effects and, as a result, a different determination of corresponding Isgur-Wise functions. Only the prediction of Ref. [6] is consistent with data (1), (2). Nevertheless, it is necessary to point out that the s quark in the final K^*

meson is not heavy enough, compared to the $\bar{\Lambda}$ parameter, which determines the scale of $1/m_Q$ corrections in heavy quark effective theory [7]. Thus the $1/m_s$ expansion is not appropriate. Notwithstanding, the ideas of heavy quark expansion can be applied to the exclusive $B \rightarrow K^*(K_2^*)\gamma$ decays. From the kinematical analysis it follows that the final $K^*(K_2^*)$ meson bears a large relativistic recoil momentum $|\Delta|$ of order of $m_b/2$ and an energy of the same order. So it is possible to expand the matrix element of the effective Hamiltonian both in inverse powers of the b quark mass for the initial state and in inverse powers of the recoil momentum $|\Delta|$ for the final state. Such an expansion has been realized by us for the $B \rightarrow K^*(892)\gamma$ decay in the framework of the relativistic quark model [8].¹ The obtained branching ratio for this decay was found in good agreement with experimental data. Here we extend this analysis to the decay $B \rightarrow K_2^*(1430)\gamma$.

In the standard model $B \rightarrow K^{**}\gamma$ decays are governed by the contribution of the electromagnetic dipole operator O_7 to the effective Hamiltonian which is obtained by integrating out the top quark and W boson and using the Wilson expansion [9]:

$$O_7 = \frac{e}{16\pi^2} \bar{s} \sigma^{\mu\nu} (m_b P_R + m_s P_L) b F_{\mu\nu}, \quad P_{R,L} = (1 \pm \gamma_5)/2. \quad (3)$$

The matrix elements of this operator between the initial B meson state and the final state of the orbitally excited tensor K_2^* meson have the following covariant decomposition

$$\begin{aligned} \langle K_2^*(p', \epsilon) | \bar{s} i k_\nu \sigma_{\mu\nu} b | B(p) \rangle &= i g_+(k^2) \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu\beta} \frac{p^\beta}{M_B} k^\lambda (p + p')^\sigma, \\ \langle K_2^*(p', \epsilon) | \bar{s} i k_\nu \sigma_{\mu\nu} \gamma_5 b | B(p) \rangle &= g_+(k^2) \left(\epsilon_{\beta\gamma}^* \frac{p^\beta p^\gamma}{M_B} (p + p')_\mu - \epsilon_{\mu\beta}^* \frac{p^\beta}{M_B} (p^2 - p'^2) \right) \\ &\quad + g_-(k^2) \left(\epsilon_{\beta\gamma}^* \frac{p^\beta p^\gamma}{M_B} k_\mu - \epsilon_{\mu\beta}^* \frac{p^\beta}{M_B} k^2 \right) \\ &\quad + h(k^2) ((p^2 - p'^2) k_\mu - (p + p')_\mu k^2) \epsilon_{\beta\gamma}^* \frac{p^\beta p^\gamma}{M_B^2 M_{K_2^*}^2}, \end{aligned} \quad (4)$$

where $\epsilon_{\mu\nu}$ is a polarization tensor of the final tensor meson and $k = p - p'$ is the four momentum of the emitted photon. The exclusive decay rate for the emission of a real photon ($k^2 = 0$) is determined by the single form factor $g_+(0)$ and is given by

$$\Gamma(B \rightarrow K_2^* \gamma) = \frac{\alpha}{256\pi^4} G_F^2 m_b^5 |V_{tb} V_{ts}|^2 |C_7(m_b)|^2 g_+^2(0) \frac{M_B^2}{M_{K_2^*}^2} \left(1 - \frac{M_{K_2^*}^2}{M_B^2} \right)^5 \left(1 + \frac{M_{K_2^*}^2}{M_B^2} \right), \quad (5)$$

where V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements and $C_7(m_b)$ is the Wilson coefficient in front of the operator O_7 . It is convenient to consider the ratio of exclusive to inclusive branching ratios, for which we have

$$R_{K_2^*} \equiv \frac{BR(B \rightarrow K_2^*(1430)\gamma)}{BR(B \rightarrow X_s \gamma)} = \frac{1}{8} g_+^2(0) \frac{M_B^2}{M_{K_2^*}^2} \frac{\left(1 - M_{K_2^*}^2/M_B^2 \right)^5 \left(1 + M_{K_2^*}^2/M_B^2 \right)}{(1 - m_s^2/m_b^2)^3 (1 + m_s^2/m_b^2)}. \quad (6)$$

¹It is important to note that rare radiative decays of B mesons require a completely relativistic treatment, because the recoil momentum of the final meson is large compared to the s quark mass.

The recent experimental value for the inclusive decay branching ratio [10]

$$BR(B \rightarrow X_s \gamma) = (3.15 \pm 0.35 \pm 0.41) \times 10^{-4}$$

is in a good agreement with theoretical calculations.

Now we use the relativistic quark model for the calculation of the form factor $g_+(0)$. In our model a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [11] of the Schrödinger type [12]:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_M(\mathbf{q}), \quad (7)$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_q^2 - m_Q^2)^2}{4M^3}. \quad (8)$$

In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_q + m_Q)^2][M^2 - (m_q - m_Q)^2]}{4M^2}. \quad (9)$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (7) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz-structure of the confining quark-antiquark interaction in the meson. In constructing the quasipotential of the quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by [13]

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) &= \bar{u}_q(p) \bar{u}_Q(-p) \mathcal{V}(\mathbf{p}, \mathbf{q}; M) u_q(q) u_Q(-q) \\ &= \bar{u}_q(p) \bar{u}_Q(-p) \left\{ \frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma_q^\mu \gamma_Q^\nu \right. \\ &\quad \left. + V_{\text{conf}}^V(\mathbf{k}) \Gamma_q^\mu \Gamma_{Q;\mu} + V_{\text{conf}}^S(\mathbf{k}) \right\} u_q(q) u_Q(-q), \end{aligned} \quad (10)$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma} \mathbf{p}}{\epsilon(p) + m} \end{pmatrix} \chi^\lambda \quad (11)$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m} \sigma_{\mu\nu} k^\nu, \quad (12)$$

where κ is the Pauli interaction constant characterizing the nonperturbative anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V_{\text{conf}}^V(r) = (1 - \varepsilon)(Ar + B), \quad V_{\text{conf}}^S(r) = \varepsilon(Ar + B), \quad (13)$$

reproducing

$$V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B, \quad (14)$$

where ε is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in v^2/c^2 , can be found in Refs. [13,14] and for heavy-light mesons in [15]. All the parameters of our model, such as quark masses, parameters of the linear confining potential, mixing coefficient ε and anomalous chromomagnetic quark moment κ , were fixed from the analysis of heavy quarkonia masses [13] and radiative decays [16]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18$ GeV² and $B = -0.30$ GeV have the usual quark model values. In Ref. [17] we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson ground states up to the second order in inverse powers of the heavy quark masses. It has been found that the general structure of the leading, first, and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential in our model. The analysis of the first order corrections [17] fixes the value of the Pauli interaction constant $\kappa = -1$. The same value of κ was found previously from the fine splitting of heavy quarkonia 3P_J -states [13]. The value of the parameter mixing vector and scalar confining potentials $\varepsilon = -1$ was found from the analysis of the second order corrections [17]. This value is very close to the one determined from considering radiative decays of heavy quarkonia [16].

In the quasipotential approach, the matrix element of the weak current $J_\mu = \bar{s}_2^i k^\nu \sigma_{\mu\nu} (1 + \gamma^5) b$ between the states of a B meson and an orbitally excited K_2^* meson has the form [18]

$$\langle K_2^* | J_\mu(0) | B \rangle = \int \frac{d^3p d^3q}{(2\pi)^6} \bar{\Psi}_{K_2^*}(\mathbf{p}) \Gamma_\mu(\mathbf{p}, \mathbf{q}) \Psi_B(\mathbf{q}), \quad (15)$$

where $\Gamma_\mu(\mathbf{p}, \mathbf{q})$ is the two-particle vertex function and Ψ_{B, K_2^*} are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ explicitly depends on the Lorentz structure of the $q\bar{q}$ -interaction. The vertex functions look like

$$\Gamma_\mu^{(1)}(\mathbf{p}, \mathbf{q}) = \bar{u}_s(p_1) \frac{i}{2} \sigma_{\mu\nu} k^\nu (1 + \gamma^5) u_b(q_1) (2\pi)^3 \delta(\mathbf{p}_2 - \mathbf{q}_2), \quad (16)$$

and

$$\begin{aligned} \Gamma_\mu^{(2)}(\mathbf{p}, \mathbf{q}) = & \bar{u}_s(p_1) \bar{u}_q(p_2) \frac{1}{2} \left\{ i\sigma_{1\mu\nu} k_\nu (1 + \gamma_1^5) \frac{\Lambda_b^{(-)}(k_1)}{\epsilon_b(k_1) + \epsilon_b(p_1)} \gamma_1^0 \mathcal{V}(\mathbf{p}_2 - \mathbf{q}_2) \right. \\ & \left. + \mathcal{V}(\mathbf{p}_2 - \mathbf{q}_2) \frac{\Lambda_s^{(-)}(k_1')}{\epsilon_s(k_1') + \epsilon_s(q_1)} \gamma_1^0 i\sigma_{1\mu\nu} k_\nu (1 + \gamma_1^5) \right\} u_b(q_1) u_q(q_2), \end{aligned} \quad (17)$$

where $\mathbf{k}_1 = \mathbf{p}_1 - \Delta$; $\mathbf{k}'_1 = \mathbf{q}_1 + \Delta$; $\Delta = \mathbf{p}_{K_2^*} - \mathbf{p}_B$;

$$\Lambda^{(-)}(p) = \frac{\epsilon(p) - (m\gamma^0 + \gamma^0(\gamma\mathbf{p}))}{2\epsilon(p)}.$$

It is important to note that the wave functions entering the weak current matrix element (15) cannot be both in the rest frame. In the B meson rest frame, the K_2^* meson is moving with the recoil momentum Δ . The wave function of the moving K_2^* meson $\Psi_{K_2^* \Delta}$ is connected with the K_2^* wave function in the rest frame $\Psi_{K_2^* \mathbf{0}} \equiv \Psi_{K_2^*}$ by the transformation [18]

$$\Psi_{K_2^* \Delta}(\mathbf{p}) = D_s^{1/2}(R_{L\Delta}^W) D_q^{1/2}(R_{L\Delta}^W) \Psi_{K_2^* \mathbf{0}}(\mathbf{p}), \quad (18)$$

where R^W is the Wigner rotation, L_Δ is the Lorentz boost from the meson rest frame to a moving one. The wave functions of B and K_2^* mesons at rest were calculated by numerical solution of the quasipotential equation (7).

We substitute the vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ given by Eqs. (16) and (17) in the decay matrix element (15) and take into account the wave function transformation (18). The resulting structure of this matrix element is rather complicated, because it is necessary to integrate both over d^3p and d^3q . The δ function in expression (16) permits us to perform one of these integrations and thus this contribution can be easily calculated. The calculation of the vertex function $\Gamma^{(2)}$ contribution is more difficult. Here, instead of a δ function, we have a complicated structure, containing the $q\bar{q}$ interaction potential in the meson. However, we can expand this contribution in the inverse powers of the heavy b quark mass and large recoil momentum $|\Delta| \sim m_b/2$ of the final K^{**} meson. Such an expansion is carried out up to the second order.² Then we use the quasipotential equation in order to perform one of the integrations in the current matrix element. As a result we get for the form factor $g_+(0)$ the following expression with $\kappa = -1$

$$g_+(0) = g_+^{(1)}(0) + (1 - \varepsilon)g_+^{(2)V}(0) + \varepsilon g_+^{(2)S}(0) \quad (19)$$

$$\begin{aligned} g_+^{(1)}(0) = & \frac{1}{\sqrt{3}} \sqrt{\frac{E_{K_2^*}}{M_B}} \frac{M_{K_2^*}}{E_{K_2^*} + M_{K_2^*}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{K_2^*} \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \\ & \times \sqrt{\frac{\epsilon_s(p + \Delta) + m_s}{2\epsilon_s(p + \Delta)}} \sqrt{\frac{\epsilon_b(p) + m_b}{2\epsilon_b(p)}} \left\{ -3(E_{K_2^*} + M_{K_2^*}) \frac{(\mathbf{p} \cdot \Delta)}{p\Delta^2} \right. \\ & \times \left[1 + \frac{M_B - E_{K_2^*}}{\epsilon_s(p + \Delta) + m_s} \right] + \left[\frac{p}{\epsilon_q(p) + m_q} - \frac{p}{\epsilon_s(p + \Delta) + m_s} \right] \\ & \times \left[1 + \frac{M_B - E_{K_2^*}}{\epsilon_s(p + \Delta) + m_s} - \frac{\mathbf{p}^2}{[\epsilon_s(p + \Delta) + m_s][\epsilon_b(p) + m_b]} \right] \Big\} \psi_B(\mathbf{p}), \end{aligned} \quad (20)$$

²This means that in expressions for $g_+^{(2)V}(0)$ and $g_+^{(2)S}(0)$ we neglect terms proportional to the third order product of small binding energies and ratios $\mathbf{p}^2/\epsilon_s^3(\Delta)$, $\mathbf{p}^2/\epsilon_b^3(\Delta)$ as well as higher order terms.

$$\begin{aligned}
g_+^{(2)V}(0) = & \frac{1}{\sqrt{3}} \sqrt{\frac{E_{K_2^*}}{M_B}} \frac{M_{K_2^*}}{E_{K_2^*} + M_{K_2^*}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{K_2^*} \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \\
& \times \sqrt{\frac{\epsilon_s(\Delta) + m_s}{2\epsilon_s(\Delta)}} \left\{ 3(E_{K_2^*} + M_{K_2^*}) \frac{(\mathbf{p} \cdot \Delta)}{p\Delta^2} \frac{M_B - \epsilon_b(p) - \epsilon_q(p)}{2[\epsilon_s(\Delta) + m_s]} - \frac{p}{\epsilon_q(p) + m_q} \right. \\
& \times \frac{1}{2[\epsilon_s(\Delta) + m_s]^2} \left((M_B + M_{K_2^*})[M_B - \epsilon_b(p) - \epsilon_q(p)] + (E_{K_2^*} + M_{K_2^*}) \right. \\
& \left. \left. \times \left[M_{K_2^*} - \epsilon_s \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) - \epsilon_s \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \right] \right) \right\} \psi_B(\mathbf{p}), \quad (21)
\end{aligned}$$

$$\begin{aligned}
g_+^{(2)S}(0) = & \frac{1}{\sqrt{3}} \sqrt{\frac{E_{K_2^*}}{M_B}} \frac{M_{K_2^*}}{E_{K_2^*} + M_{K_2^*}} \int \frac{d^3p}{(2\pi)^3} \bar{\psi}_{K_2^*} \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \\
& \times \sqrt{\frac{\epsilon_s(\Delta) + m_s}{2\epsilon_s(\Delta)}} \left\{ \left[-3(E_{K_2^*} + M_{K_2^*}) \frac{(\mathbf{p} \cdot \Delta)}{p\Delta^2} + \frac{p}{\epsilon_q(p) + m_q} \right] \right. \\
& \times \left(\frac{\epsilon_s(\Delta) - m_s}{2\epsilon_s(\Delta)[\epsilon_s(\Delta) + m_s]} \left[M_{K_2^*} - \epsilon_s \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \right. \right. \\
& \left. \left. - \epsilon_s \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \right] + \frac{M_B - E_{K_2^*}}{2[\epsilon_s(p + \Delta) + m_s]^2} \left[M_B + M_{K_2^*} - \epsilon_b(p) - \epsilon_q(p) \right. \right. \\
& \left. \left. - \epsilon_s \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) - \epsilon_s \left(\mathbf{p} + \frac{2\epsilon_q}{E_{K_2^*} + M_{K_2^*}} \Delta \right) \right] \right) \right\} \psi_B(\mathbf{p}), \quad (22)
\end{aligned}$$

where the superscripts “(1)” and “(2)” correspond to contributions coming from Figs. 1 and 2, S and V mean the scalar and vector potentials in Eq. (13), $\psi_{K_2^*,B}$ are radial parts of the wave functions. The recoil momentum and the energy of the K_2^* meson are given by

$$|\Delta| = \frac{M_B^2 - M_{K_2^*}^2}{2M_B}; \quad E_{K_2^*} = \frac{M_B^2 + M_{K_2^*}^2}{2M_B}. \quad (23)$$

We can check the consistency of our resulting formulas by taking the formal limit of b and s quark masses going to infinity.³ In this limit according to the heavy quark effective theory [19] the function $\xi_F = 2\sqrt{M_B M_{K_2^*}} g_+ / (M_B + M_{K_2^*})$ should coincide with the Isgur-Wise function τ for the semileptonic B decay to the orbitally excited tensor D meson, $B \rightarrow D_2^* e \nu$. Such semileptonic decays have been considered by us in Ref. [20]. It is easy to verify that the equality of ξ_F and τ is satisfied in our model if we also use the expansion in $(w-1)/(w+1)$ (w is a scalar product of four-velocities of the initial and final mesons), which is small for the $B \rightarrow D_2^* e \nu$ decay [20]. Using Eq. (19) to calculate the ratio of the form factor $g_+(0)$ in the infinitely heavy b and s quark limit to the same form factor in the leading order of expansions in inverse powers of the heavy b quark mass and large recoil momentum $|\Delta|$ we find that it is equal to $M_B / \sqrt{M_B^2 + M_{K_2^*}^2} \approx 0.965$. The corresponding ratio of form factors of the exclusive rare radiative B decay to the vector K^* meson $F_1(0)$ (see Eq. (23) of Ref. [8]) is equal to $M_B / \sqrt{M_B^2 + M_{K^*}^2} \approx 0.986$. Therefore we conclude that the

³As it was noted above such limit is justified only for the b quark.

ratios of form factors $g_+(0)/F_1(0)$ in the leading order of these expansions differ by factor $\sqrt{M_B^2 + M_{K^*}^2}/\sqrt{M_B^2 + M_{K_2^*}^2} \approx 0.98$. This is the consequence of the relativistic dynamics leading to the effective expansion in inverse powers of the s quark energy $\epsilon_s(p + \Delta) = \sqrt{(\mathbf{p} + \Delta)^2 + m_s^2}$, which is large in one case due to the large s quark mass and in the other one due to the large recoil momentum Δ . As a result both expansions give similar final expressions in the leading order. Thus we can expect that the ratio r from (2) in our calculations should be close to the one found in the infinitely heavy s quark limit [6].

The results of numerical calculations using formulas (5), (6), (19)–(22) for $\varepsilon = -1$ are given in Table I. There we also show our previous predictions for the $B \rightarrow K^*\gamma$ decay [8]. Our results are confronted with other theoretical calculations [4–6] and recent experimental data [2]. We find a good agreement of our predictions for decay rates with the experiment and estimates of Ref. [6]. Other theoretical calculations substantially disagree with data either for $B \rightarrow K^*\gamma$ [4] or for $B \rightarrow K_2^*\gamma$ [5] decay rates. As a result our predictions and those of Ref. [6] for the ratio r from (2) are well consistent with experiment, while the r estimates of [4] and [5] are several times larger than the experimental value (see Table I). As it was argued above, it is not accidental that r values in our and Ref. [6] approaches are close. The agreement of both predictions for branching ratios could be explained by some specific cancellation of finite s quark mass effects and relativistic corrections which were neglected in Ref. [6]. Though our numerical results agree with Ref. [6], we believe that our analysis is more consistent and reliable. We do not use the ill-defined limit $m_s \rightarrow \infty$, and our quark model consistently takes into account some important relativistic effects, for example, the Lorentz transformation of the wave function of the final K^{**} meson (see Eq. (18)). Such a transformation turns out to be very important, especially for B decays to orbitally excited mesons [20]. The large value of the recoil momentum $|\Delta| \sim m_b/2$ makes relativistic effects to play a significant role. On the other hand this fact simplifies our analysis since it allows to make an expansion both in inverse powers of the large b quark mass and in the large recoil momentum.

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REFERENCES

- [1] CLEO Collaboration, R. Ammar et al., Phys. Rev. Lett. 71 (1993) 674.
- [2] CLEO Collaboration, T. E. Coan et al., Phys.Rev. Lett. 84 (2000) 5283.
- [3] K. Lingel, T. Skwarnicki and J. G. Smith, Ann. Rev. Nucl. Part. Sci. 48 (1998) 253.
- [4] T. Altomari, Phys. Rev. D 37 (1988) 677.
- [5] A. Ali, T. Ohl and T. Mannel, Phys. Lett. B 298 (1993) 195.
- [6] S. Veseli and M. G. Olsson, Phys. Lett. B 367 (1996) 309.
- [7] M. Neubert, Phys. Rep. 245 (1994) 259.
- [8] R. N. Faustov and V. O. Galkin, Phys. Rev. D 52 (1995) 5131.
- [9] B. Grinstein, R. Springer and M. Wise, Phys. Lett. B 202 (1988) 138; Nucl. Phys. B 339 (1990) 269; R. Grigjanis, P. J. O'Donnell, M. Sutherland and H. Navelet, Phys. Lett. B 237 (1990) 252; G. Cella, G. Curci, G. Ricciardi and A. Vicere, Phys. Lett. B 248 (1990) 181; B 325 (1994) 227; M. Misiak, Phys. Lett. B 269 (1991) 161; B 321 (1994) 193; Nucl. Phys. B 393 (1993) 23.
- [10] J. Trampetić, hep-ph/0002131.
- [11] A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento 29 (1963) 380.
- [12] A. P. Martynenko and R. N. Faustov, Teor. Mat. Fiz. 64 (1985) 179 [Theor. Math. Phys. 64 (1985) 765].
- [13] V.O. Galkin, A. Yu. Mishurov and R. N. Faustov, Yad. Fiz. 55 (1992) 2175 [Sov. J. Nucl. Phys. 55 (1992) 1207]; D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 62 (2000) 034014.
- [14] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 7 (1999) 539.
- [15] D. Ebert, V. O. Galkin and R. N. Faustov, Phys. Rev. D 57 (1998) 5663; 59 (1999) 019902 (Erratum).
- [16] V. O. Galkin and R. N. Faustov, Yad. Fiz. 44 (1986) 1575 [Sov. J. Nucl. Phys. 44 (1986) 1023]; V. O. Galkin, A. Yu. Mishurov and R. N. Faustov, Yad. Fiz. 51 (1990) 1101 [Sov. J. Nucl. Phys. 51 (1990) 705].
- [17] R. N. Faustov and V. O. Galkin, Z. Phys. C 66 (1995) 119; D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 62 (2000) 014032.
- [18] R. N. Faustov, Ann. Phys. 78 (1973) 176; Nuovo Cimento A 69 (1970) 37.
- [19] S. Veseli and M. G. Olsson, Phys. Lett. B 367 (1996) 302.
- [20] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 61 (2000) 014016.

TABLES

TABLE I. Our results in comparison with other theoretical predictions and experimental data for branching ratios and their ratios $R_{K^*} \equiv \frac{BR(B \rightarrow K^* \gamma)}{BR(B \rightarrow X_s \gamma)}$, $R_{K_2^*} \equiv \frac{BR(B \rightarrow K_2^* \gamma)}{BR(B \rightarrow X_s \gamma)}$, $r \equiv \frac{BR(B \rightarrow K_2^* \gamma)}{BR(B \rightarrow K^* \gamma)}$. Our values for the $B \rightarrow K^* \gamma$ decay are taken from Ref. [8].

Value	our	Ref. [4]	Ref. [5]	Ref. [6]	Exp. [2]
$BR(B \rightarrow K^* \gamma) \times 10^5$	4.5 ± 1.5	1.35	$1.4 - 4.9$	4.71 ± 1.79	$4.55^{+0.72}_{-0.68} \pm 0.34^a$ $3.76^{+0.89}_{-0.83} \pm 0.28^b$
$R_{K^*} (\%)$	15 ± 3	4.5	$3.5 - 12.2$	16.8 ± 6.4	
$BR(B \rightarrow K_2^* \gamma) \times 10^5$	1.7 ± 0.6	1.8	$6.9 - 14.8$	1.73 ± 0.80	$1.66^{+0.59}_{-0.53} \pm 0.13$
$R_{K_2^*} (\%)$	5.7 ± 1.2	6.0	$17.3 - 37.1$	6.2 ± 2.9	
r	0.38 ± 0.08	1.3	$3.0 - 4.9$	0.37 ± 0.10	$0.39^{+0.15}_{-0.13}$

^a $B^0 \rightarrow K^{*0} \gamma$

^b $B^+ \rightarrow K^{*+} \gamma$

FIGURES

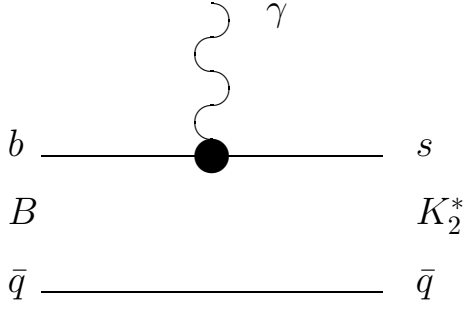


FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ corresponding to Eq. (16).

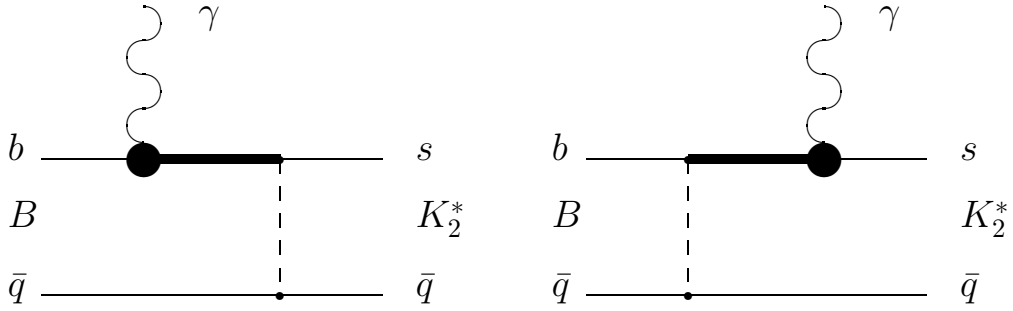


FIG. 2. Vertex function $\Gamma^{(2)}$ corresponding to Eq. (17). Dashed lines represent the effective potential \mathcal{V} in Eq. (10). Bold lines denote the negative-energy part of the quark propagator.